Efficient Modeling Procedure of Novel Grating Tiling Device Using Multibody System Approach

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Abstract. This paper proposes a multibody system (MBS) procedure for a novel aperture grating device which considered as a rigid-flexible multibody system. The MBS model is constructed based on the load assumptions due to grating movement. This movement can be utilized in laser generation and its consequent applications involve precision measuring instruments, optical communication and many other applications. The MBS model is used to estimate the system accelerations, static as well as dynamic loads based on the obtained Lagrange multipliers. According to the dynamic behavior and the generated forces, the mechanical design process of the grating device can be implemented with trade offs optimization in terms of grating parameters. The numerical manipulations of a proposed grating device are presented using MATLAB symbolic toolbox with very good results regarding the positioning precision, stability and design specifications.

Keywords: Multibody dynamics · Rigid-Flexible systems · Grating device system

1 Introduction

The grating device of high inertia and supremely precision is the core element for optical devices of observational instruments which are applied in various fields[8]. Recently, research and development have made progress in the areas of optimization, modeling, simulation and design of grating devices [12]. The structure analysis, the kinematics and dynamics analysis of large aperture grating device including both the macro and/or micro-drive control mechanism are important parameters to optimize grating structure [3]. Several methods used for the modeling, design, control and analysis of the grating devices are carried out [6], in which the dynamics model is based on finite element analysis (FEA). FEA can be considered more suitable for modeling elastic bodies subjected to small deformation behavior, while the grating devices show definite rigid body

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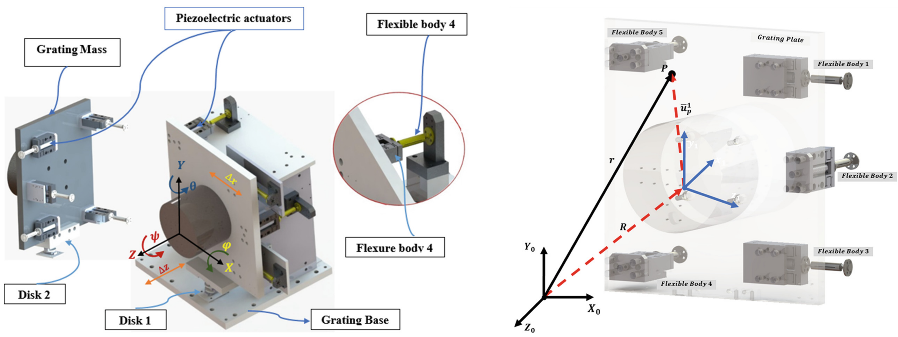
motion and deformable motion of the body as well. Multibody system dynamics became the most economical venue in product design and optimization of complex mechanical systems. [11] shows that the MBS dynamics is the most suitable technique to model those systems that show definite rotations as well as small deformation. The main parameters affecting the stability of the grating device are system structure and control method [10]. Further improvement of the stability and design of the grating device can be achieved by implementing the multibody system approach in order to involve what other methods ignore. This paper is organized as follows: Sect. 2 introduces the mechanical grating device as a multibody system. In Sect. 3, the constraint's function is expressed, and the model is constructed. Section 4 presents the simulation results that are followed by the conclusion in Sect. 5.

2 Kinematics of Grating Device Motion

The grating device shown in Fig. 1-a can easily be identified as a typical parallel mechanism [6]. This mechanism includes two transnational joints and one spherical joint lateral shift X_1, longitudinal piston Z_1, angular tip θ, angular tilt ϕ and linear rotation ψ. Most of grating device applications require that the tiling accuracy of translation and rotation in the millimeter range be kept on the micro-radial and nano-meter scale and be maintained stably for a long time. The grating device, as a multibody system consists of a base, disk1, disk2, grating mass, five flexure bodies and five flexible bodies considered as an elastic elements for the movement of piezoelectric actuators, see Fig. 1-a. Defining the coordinate system as (X_0, Y_0, Z_0) as the global frame that is fixed in time [7]. The local coordinates of an arbitrary point $\mathbf{P}$ lie on the grating mass (*body 4*) with respect to the body frame (x_4, y_4, z_4) can be described by the vectors $\bar{\mathbf{u}}_r^i$ for rigid bodies and $\bar{\mathbf{u}}_f^i$ for flexible bodies while the global position $\mathbf{r}$, define with respect to the global coordinates can be expressed as [1]:



(a) Grating device system (b) Point P define on grating mass

Fig. 1. Grating device as multibody system

$$\mathbf{r}^i = \mathbf{R}^i + \mathbf{A}^i \; (\bar{\mathbf{u}}_r^i + \bar{\mathbf{u}}_f^i) \tag{1}$$

where $\mathbf{r}^i = [r_x^i, r_y^i, r_z^i]^T$, is the global position of an arbitrary point, $\mathbf{R}^i = [R_x^i, R_y^i, R_z^i]^T$, is the global position of the origin of the grating coordinate system, and $\mathbf{A}^i$ is the transformation matrix function on generalized coordinate system, see Fig. 1-b. It is clear from Eq. (1) that the global position vector of an arbitrary point on the body coordinate system can be written in terms of the rotational coordinate of the body, $\theta^i = [\phi^i, \theta^i, \psi^i]^T$, as well as the translation of the frame-origin of the body, $\mathbf{R}^i$. That is, the most general displacement can be described by a translation of a reference point plus a rotation about an axis passing through this point [2]. In the floating frame of reference formulation, the deformation of the flexible body can be defined concerning its reference [5]. Multibody systems with rigid and flexible bodies are subjected to kinematic constraints resulting from mechanical joints. The constraints equation can be written as:

$$\mathbf{C}(\mathbf{q}_r, \mathbf{q}_f, t) = \mathbf{0} \tag{2}$$

where $\mathbf{C}$ is the vector of constraint functions, $\mathbf{q}_r$ is the vector of rigid bodies coordinates, $\mathbf{q}_f$ is the vector of flexible bodies coordinates and t is time. The equations that govern the dynamics of a multibody system can be systematically obtained as [1]:

$$\begin{bmatrix} \mathbf{M}_{rr} & \mathbf{M}_{rf} & \mathbf{C}_{q_r}^T \\ \mathbf{M}_{fr} & \mathbf{M}_{ff} & \mathbf{C}_{q_f}^T \\ \mathbf{C}_{q_r} & \mathbf{C}_{q_f} & \mathbf{0} \end{bmatrix} \begin{bmatrix} \ddot{\mathbf{q}}_\mathbf{r} \\ \ddot{\mathbf{q}}_\mathbf{f} \\ \lambda \end{bmatrix} = \begin{bmatrix} \mathbf{Q}_r \\ \mathbf{Q}_f \\ \mathbf{Q}_d \end{bmatrix} \tag{3}$$

where $\mathbf{M_{rr}}$ is the system mass matrix associated with rigid bodies, $\mathbf{M_{ff}}$ is the system mass matrix associated with flexible bodies, The dynamic coupling between the rigid body motion and the flexible body deformation is represented by the two matrices $\mathbf{M_{rf}}$ and $\mathbf{M_{fr}}$. The matrices $\mathbf{C}_{\mathbf{q}r}$ and $\mathbf{C}_{\mathbf{q}f}$ are the Jacobian matrices of the kinematic constraints function $\mathbf{C}$, λ is the vector of Lagrange multipliers and $\mathbf{Q}_d$ is the vector that absorbs all quadratic terms of velocity and $\mathbf{Q}$ is the vector of externally applied forces. Equation 3 yields a system of differential algebraic equations (DAE). The vector $\ddot{\mathbf{q}}_\mathbf{r}$ and $\ddot{\mathbf{q}}_\mathbf{f}$ can be integrated to determine the coordinates and velocities for rigid and flexible bodies respectively. The vector λ can be used to determine the generalized reaction forces that used for initiating the design process.

3 Multibody Model of Grating System

The model of the grating system shown in Fig. 1, can be constructed, without loss of generality, as shown in Table 1. Based on the application, the movement grating mass can be described in spatial with five movement translation on X and Z and rotational about three axes with Euler angles ϕ, θ and ψ. Let the contact point $\mathbf{P}$ is located on the grating mass frame. The system of generalized coordinates denoted by $\mathbf{q}_r$ and can define function of Euler angels as:

Table 1. Components of Grating device system

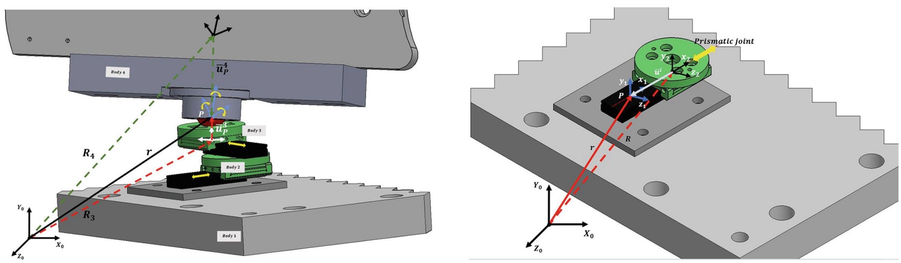
Joint Type	Body(i)	Body(j)
Fixed	Grating Base	Ground
Prismatic(X)	$Disk_1$	Grating Base
Prismatic(Z)	$Disk_2$	$Disk_1$
Spherical	Grating mass	$Disk_2$
Fixed	Flexure bodies	Grating mass
Fixed	Flexible bodies	Flexure bodies
Fixed	Flexible bodies	Grating base

$$\mathbf{q}_r^1 = \left[q_1^1 \; q_2^1 \; q_3^1 \; \phi^1 \; \theta^1 \; \psi^1 \right] \tag{4}$$

where ${q_r}^1, {q_r}^2, ..., {q_r}^{14}$ are local coordinates system for each body in grating device model define in translation and rotational coordinates. The vector of elastic bodies coordinates $\mathbf{q_f}$ can be expressed in terms of the modal coordinates and equal zero in the case of a rigid body displacement[4]. The constraints equations of the grating device which considered as holonomic constraints $\mathbf{C}^h$ can be obtained using multibody constraints equation of Spherical, Prismatic and Rigid joints. Figure 2-a shows a spherical joint between grating mass and disk2. The kinematic constraints of the spherical joint can be written as:

$$\mathbf{C}^h(q^3, q^4, t) = R^3 + A^3\bar{u}_3^i - R^4 - A^4\bar{u}_p^4 = 0 \tag{5}$$

where $\mathbf{R}^3$ and $\mathbf{R}^4$ are the global position vectors of the origins of the coordinate systems of bodies 3 and 4 respectively. $\mathbf{A}^3$ and $\mathbf{A}^4$ are the transformation matrices of the two bodies connected and expressed a function on orientation coordinates, $\bar{\mathbf{u}}_p^3$ and $\bar{\mathbf{u}}_p^4$ are the local position vectors of the joint. Constraints equation of spherical joint allows the grating mass to rotate around three axes with ϕ, θ and ψ, see Fig. 1-a . Figure 2-b shows a prismatic joint between base and disk1 which allows grating mass translation in Z direction. Constraints equations for other rigid and prismatic joints can be defined similarly to spherical



(a) Spherical joint (b) prismatic joint

Fig. 2. Holonomic multibody joints

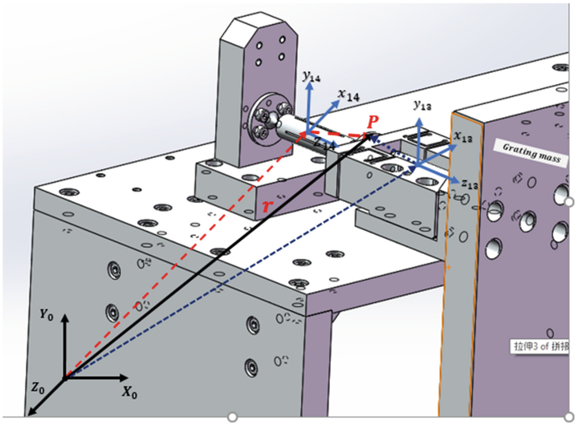


Fig. 3. Rigid joint between flexible body4 and flexure body4

joints [2]. Using the kinematic description of the Floating Frame of Reference formulation FRR, the vector of constraints equation of a flexible body can be expressed. Figure 3 shows a fixed joint between flexible body5 with flexure body5. Symbolic computer procedures can be used to compute constraints equation [9]. Other equation of motion items can be computed as [2]. The solution of Eq. 3 presented in the preceding section defines the vectors of acceleration and Lagrange multipliers which can be used to determine the vector of generalized constraint forces.

4 Numerical Simulation

In this section, the mathematical model of a small-scale grating device is constructed using the multibody dynamics approach based on the Lagrange formulation described in the previous section using MATLAB symbolic toolbox. Simulation results that include kinematics and dynamics results are shown. The physical parameters are listed in Table 2 define the initial parameters of the grating system. The adjustment of the grating relies on the collective effect of the five piezoelectric actuators directly act on grating mass through flexure bodies. The external applied forces due to piezoelectric actuators are F = 4700 N, see Fig. 1-a. By applying external forces, the corresponding structural displacement of the system can be obtained by controlling movements of piezoelectric actuators. In literature, various types of numerical integrators can be used to solve the resulting DAEs of multibody system. Among of these methods are: Rung-Kutta, Adams-Bashforth, Newmark and HHT Method. Explicit and/or implicit integrators can be implemented according to the status of numerical stiffness of the integration process and the number of coordinates (full coordinates integration or reduced order integration). As presented in [4], the multi-step - explicit (Adams-Bashforth) method is the most suited integrator for rigid and/or flexible bodies with small deformation problems in the case of full-coordinates integration. Table 3 illustrates maximum grating parameters the system can achieve with applied forces. These values are similar to those found in [10].

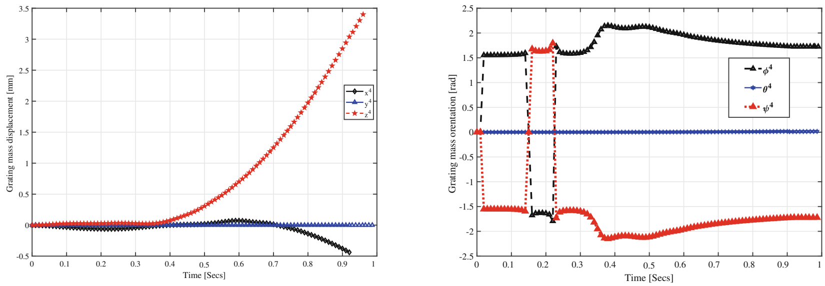
Table 2. Grating device parameters employed in the numerical simulation

Components	Mass (kg)	$I_{xx}(Kg.m^2)$	$I_{yy}(Kg.m^2)$	$I_{zz}(Kg.m^2)$
Grating base	86.34	4.92	3.64	2.8
$Disk_1$	0.15	0.000035	0.000035	0.000018
$Disk_2$	0.11	0.000024	0.000013	0.000013
Grating mass	78.26	1.8	1.3	0.985
Flexure part	0.068	0.0001	0.0001	0.0000025
Flexible part	0.012	0.00001	0.00001	0.00000063

Table 3. Maximum Displacement of DOF

DOF	X(mm)	Z(mm)	$\phi(rad)$	$\theta(rad)$	$\psi(rad)$
-	±1.5	±3.00	±2.5	±1.5	±2.5

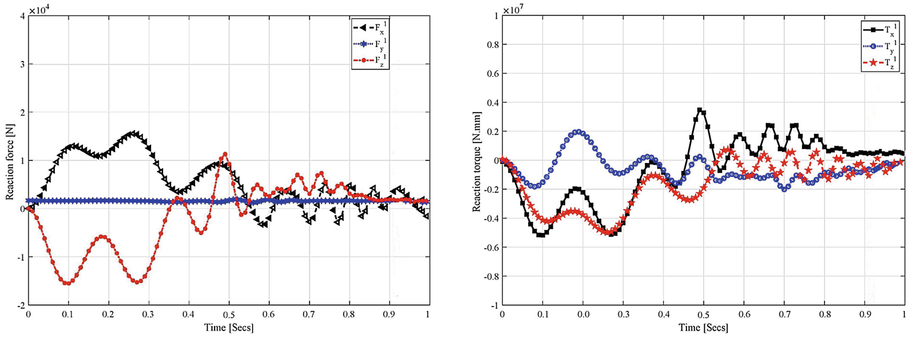
Figure 4-a shows the global position vector of the point P that coincides with the grating mass frame. The translation along Z-axis reaches about 3 mm. The translation along X-axis 0.5 mm at the same forces while there is no movement along Y-axis and that indicates the formulation accuracy of multibody constraints equation. Figure 4-b shows the orientation of the point. In order to establish design process, multibody model can be used to compute all dynamic reaction forces acting on system bodies. As example Fig. 5-a shows reactions



(a) Displacement of the point

(b) Orientation of the point

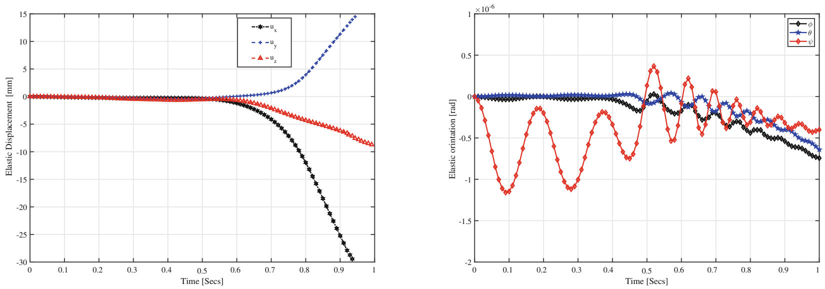
Fig. 4. Position of point P located on grating mass



(a) Reaction forces

(b) Reaction torques

Fig. 5. Reaction forces acting on grating base



(a) Translation of local frame

(b) Orientation of local frame

Fig. 6. Displacement of the flexible body4

forces acting on grating base, the values of the forces in X-axis and Z-axis are swinging due to dynamic affect and the only reaction force in Y-axis due to grating mass. Figure 5-b shows reactions moments acting on grating base. All other reactions forces acting on system joints can be computed. The flexure and other flexible bodies in the system are considered in the design of the tiling system to ensure the to system stability. Flexible bodies that exhibit valuable elasticity behavior, restrict the movement of the system and the available degrees of freedom. Figure 6 shows the displacement and orientation of the local frame of body 5. The values of dynamic reaction forces and moments computed from multibody model are similar as ones presented in [6], in which the ANSYS software is utilized. These forces represented in [6], in terms of the identification of stiffness values along the various axes, by dividing the reaction loading by its deformation output, the stiffness could be identified. The estimated torques results, both static and dynamic torques can be used to optimally design the mechanical structure of grating tiling device, and therefore, control precisely the movements of grating mass. This computational models is important in all phases of system synthesis, analysis, design and control of the grating system.

The multibody model will be used later for parameters identification of grating device.

5 Conclusion

In this paper, a systematic procedure based on multibody system approach is implemented in the modelling and simulation of novel grating tiling device. The symbolic as well as computational work has been carried out using MATLAB software. The symbolic derivation is carried out and the inherent equations of motion of the grating system has been derived. The solution of the equations of motion involve the system generalized coordinates and the associated Lagrange multipliers as well. These multipliers can be used to estimate the generalized forces and can be utilized in the design procedure of the system. Based on the simulation work of the multibody model, the optimal design of such grating devices can be established for maximum ranges of grating parameters. The obtained design of the grating device meets the requirements of high stability and can be used for high precision applications.

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